Building Functions Worksheet 1 - Answers

1. Write a symbolic function describing the distance traveled by sailboat as a function of headwind and forward speed.

\[
\text{Distance} = (\text{speed} - \text{headwind}) \times \text{time traveled}
\]

2. Kinetic energy can be written as \( KE = \frac{1}{2}mv^2 \). Write \( KE \) in terms of time \( t \).

\[
KE = \frac{1}{2}m(\frac{x}{t})^2 = \frac{mx^2}{2t^2}.
\]

3. Build a function that models the temperature of a cooling body given that the initial temperature is \( T_i \) and the rate of cooling is \( e^{-Tt} \).

\[
T(t) = T_i e^{-Tt}
\]

4. If your initial position is \( x_i \) and you are traveling at a constant speed \( v \). What is your position in terms of time \( t \)?

\[
x(t) = x_i + vt.
\]

5. If \( f(x) = x^2 + 2x + 1 \) and \( g(f) = \frac{f + \sqrt{f}}{2f} \), then what is \( g(f(x)) \)?

\[
g(f(x)) = \frac{x+2}{2x+2}.
\]

6. The likelihood of a Justin Bieber encounter is inversely proportional to how far away from his house you are. If the function is given as \( B(d) = d^n \), where \( d \) is the distance away from his house, what limitations would you expect constant \( n \) to have?

We’d expect \( n \) to have a negative value.

7. Your lifelong dream is to meet Lady Gaga in person. After seeing her in concert, you’re 100% certain this dream will come true, but for every year that you don’t see her live in concert, your half as sure that it’ll really happen. Come up with a function that describes the yearly decay of your lifelong dream.

\[
D(t) = 100 \times (\frac{1}{2})^t \text{ where } D(t) \text{ is the certainty at which you believe your dream will come true, and } t \text{ is the number of years that pass after } \text{not seeing Lady Gaga in concert.}
\]

8. The potential energy of an object is given by \( PE = mgh \). If the object is thrown and its height \( h \) is a function of time such that \( h(t) = -0.6t^2 + 3t \), what is the potential energy of the object in terms of time?

\[
PE = mgh(t) = mg(-0.6t^2 + 3t).
\]

9. Using the function you came up with, at which point in time is \( PE \) the highest? What is this value in terms of \( m \) and \( g^2 \)?

\[
t = 2.5, \ PE = mgt(3 - 0.6t) = 3.75mg.
\]

10. If the overall energy of the object is the sum of its kinetic and potential energy \( (KE = \frac{1}{2}mv^2) \), and the velocity \( v \) of the object is a function of time such that \( v = \frac{h}{t} \), what is the overall energy of the object in terms of \( m \), \( g \), and \( t \)?

\[
E = mgt(3 - 0.6t) + \frac{1}{2}m(3 - 0.6t)^2.
\]