Properties of Definite Integrals - Answer Key

1. If $\int_3^7 (f(x) - 4x)dx = 17$, find $\int_3^7 f(x)dx$.

   Ans: 97

2. If $\int_{-1}^2 f(x)dx = a$ and $\int_{-1}^2 g(x)dx = b$, find $\int_{-1}^2 (2f(x) - 3g(x) + 5)dx$.

   Ans: $2a - 3b + 8$

3. If $\int_{-a}^a f(x)^2 dx = 9$ and $\int_{-a}^a g(x)^2 dx = 5$, find $\int_{-a}^a (f(x) + g(x))^2 dx$.

   Ans: The limit cannot be determined.

4. If $g(x)$ is an odd function and $\int_{-2}^4 (2g(x) + 3x)dx = 15$, find $\int_2^4 g(x)dx$.

   Ans: $\frac{-3}{2}$

5. If $f(x)$ is even and $\int_{-2}^0 f(x)dx = \alpha$, $\int_2^5 f(x)dx = -1$, find $\left(\int_{-2}^5 f(x)dx\right)^2$.

   Ans: $4\alpha^2 - 4\alpha + 1$

6. Find the average value of $f(x) = \sqrt{9 - x^2}$ on $(-3, 3)$.

   Ans: $\frac{3\pi}{4}$

7. Find $\int_0^4 f(x)dx$ if $f(x)$ is even and $g(x)$ is odd and $\int_{-4}^4 (2f(x) + 3g(x))dx = 16$.

   Ans: 4

8. Is $f(x)$ an even or odd function so that $\int_{-b}^a f(x)dx = -\int_b^{-a} f(x)dx$ if true?

   Ans: Even

9. Find the average value of $f(x) = 5x^5$ on $[-8, 8]$.

   Ans: 0

10. If $1 \leq f(x) \leq 2x$ for $0 < x < 4$, find the bounds on average value of $f(x)$ on $[0, 4]$.

    Ans: $1 \leq$ Average value ≤ 4
