Expressing Geometric Properties Worksheet 1

1. Prove that the shape with corners $A(4, 7)$, $B(8, 5)$ and $C(10, 10)$ is a scalene triangle.
   $AB = 2\sqrt{5}$, $BC = \sqrt{29}$, $AC = 3\sqrt{5}$.

2. Prove that the shape with corners $A(1, 4)$, $B(3, 0)$, $C(1, -4)$ and $D(-1, 0)$ is a rhombus.
   $AB = BC = CD = DA = 2\sqrt{5}$.

3. Prove that the shape with corners $A(10, 8)$, $B(12, 7)$, $C(9, 6)$ and $D(11, 5)$ is a rectangle.
   $m_1 = \text{Slope}_{AB} = \text{Slope}_{CD} = -0.5$, $m_2 = \text{Slope}_{BD} = \text{Slope}_{AC} = 2$ and $m_1 m_2 = -1$.

4. Prove that the shape with corners $A(10, 8)$, $B(13, 5, 5)$ and $C(17, 12)$ is an isosceles triangle.
   $AC = BC = \sqrt{67}$ and $AB = 3\sqrt{2}$.

5. Prove that the shape with corners $A(-6, 4)$, $B(-3, -1)$, $C(0, -2)$ and $D(-6, -8)$ is a trapezoid.
   The opposite sides AB and CD are parallel, $\text{Slope}_{BP} = \text{Slope}_{CD} = 1$ whereas sides BC and AD are not parallel.

6. Prove that the shape with corners $A(14, 11)$, $B(24, 9)$, $C(26, 3)$ and $D(16, 5)$ is a parallelogram.
   Midpoint of the diagonal AC=Midpoint of the diagonal BD=$(15, 7)$, therefore tha diagonals bisect each other.

7. Prove that the shape with corners $A(8, 9)$, $B(13, 10)$, $C(14, 5)$ and $D(9, 4)$ is a square.
   $m_1 = \text{Slope}_{AB} = \text{Slope}_{CD} = 0.2$, $m_2 = \text{Slope}_{BC} = \text{Slope}_{DA} = -5$ and $m_1 m_2 = -1$, therefore adjacent sides are perpendicular. The sides are equal $AB = BC = CD = DA = \sqrt{26}$.

8. Prove that the point $(4, 11)$ lies in the line $y = 0.5x + 9$.
   Plugging $x = 4$ in the equation $0.5(4) + 9 = 11$.

9. Prove that the point $(6, 8)$ lies on the circle with a center of $(3, 4)$ and a radius 5.
   Plugging $x = 6$ and $y = 8$ in the equation of the circle is $(x-3)^2 + (y-4)^2 = (5)^2$, $(6-3)^2 + (8-4)^2 = 3^2 + 4^2 = 25$.

10. Prove that the point $(6, 8)$ lies on the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{16} = 1.$.
    Since the simplified statement of 1=1 is true, we know that (1, 6) is a set of coordinates that satisfies the equation, and therefore lies on the ellipse.