# Handout 3: Circular Logic
# Answers

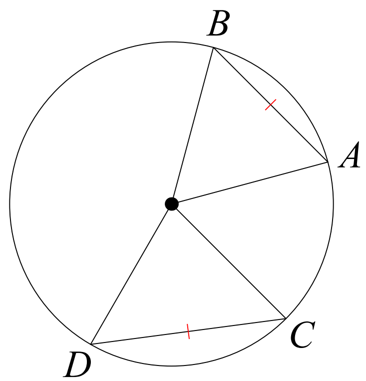

Figure 1

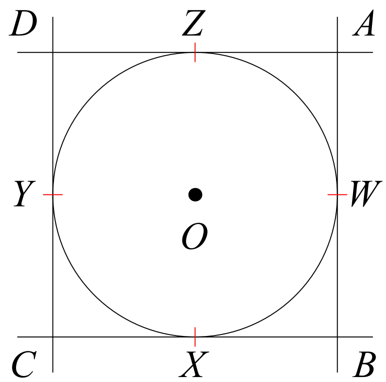

Figure 2

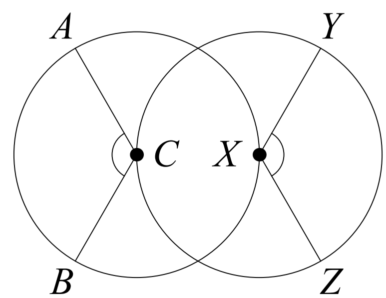

Figure 3

Questions 1-2 refer to Figure 1, where points $A$, $B$, $C$, and $D$ are on $\odot O$ and arc $AB$ is congruent to arc $CD$.

1. Prove that $\angle COD$ is congruent to $\angle AOB$.

   We are given that arc $AB$ is congruent to arc $CD$, and by definition of arc measure, $\angle AOB$ is congruent to $\angle COD$.

2. Prove that $\Delta AOB$ is congruent to $\Delta COD$.

   Since $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$ are all congruent because they're all radii and $\angle AOB$ is congruent to $\angle COD$, we have enough to say that $\Delta AOB$ is congruent to $\Delta COD$ by SAS.

Questions 3-5 refer to figure 2, where segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to $\odot O$ at $W$, $X$, $Y$, and $Z$, respectively.

3. Which theorem proves that $\overline{AB}$ and $\overline{DC}$ are both perpendicular to $\overline{WY}$, and that $\overline{AD}$ and $\overline{BC}$ are both perpendicular to $\overline{XZ}$?

   The Perpendicular Tangent Theorem.

4. Using this information, prove that $ABCD$ is a square.

   Since the distance between two parallel lines never changes, we can state that $AB = BC = XZ = WY = CD = AD$. Therefore $ABCD$ is a quadrilateral with four sides of equal length and two sets of parallel sides, which makes it a square.

Questions 6-7 refer to Figure 3, where points $A$, $B$, and $X$ are on $\odot C$ and points $C$, $Y$, and $Z$ are on $\odot X$. Also, $\angle ACB$ is congruent to $\angle ZXY$.

6. Prove that $\odot C$ and $\odot X$ are congruent.

   If we draw segment $\overline{CX}$, we see that it is a radius of both $\odot C$ and $\odot X$. Therefore $\odot C$ and $\odot X$ are congruent.

7. Prove that $\Delta ABC$ is congruent to $\Delta XYZ$.

   By definition of congruent circles, $\overline{XZ}$, $\overline{XY}$, $\overline{AC}$ and $\overline{BC}$ are all congruent. We are given that $\angle ZXY$ is congruent to $\angle ACB$, so we can state that $\Delta ACB$ is congruent to $\Delta XYZ$ by SAS.

For questions 8-10, disprove the statement by finding a counterexample.

8. Given any line segment $\overline{AB}$, you can draw exactly one circle such that $\overline{AB}$ is a chord of that circle.

   Suppose two circles intersect at points $A$ and $B$ only. (One of many possible answers.)

9. Given any line segment $\overline{AB}$, you can draw exactly one circle such that $\overline{AB}$ is a radius of that circle.

   Draw $\odot A$ containing $B$ and $\odot B$ containing $A$.

10. If two different lines $m$ and $n$ are both tangent to $\odot O$, then $m$ and $n$ must intersect at some point $P$.

    Suppose $\overline{PQ}$ is a diameter of $\odot O$, $m$ contains $P$, and $n$ contains $Q$.