Building Functions Worksheet 1 - Answers

1. Write a symbolic function describing the distance traveled by sailboat as a function of headwind and forward speed.
   Distance = (speed − headwind) × time traveled

2. Kinetic energy can be written as $KE = \frac{1}{2}mv^2$. Write KE in terms of time t.
   $KE = \frac{1}{2}m(\frac{x}{t})^2 = \frac{mx^2}{2t^2}$.

3. Build a function that models the temperature of a cooling body given that the initial temperature is T_i and the rate of cooling is e^{-Tt}.
   $T(t) = T_i e^{-Tt}$

4. If your initial position is x_i and you are traveling at a constant speed v. What is your position in terms of time t?
   $x(t) = x_i + vt$.

5. If $f(x) = x^2 + 2x + 1$ and $g(f) = \frac{f+\sqrt{f}}{2f}$, then what is $g(f(x))$?

   $g(f(x)) = \frac{x+2}{2x+2}$.

6. The likelihood of a Justin Bieber encounter is inversely proportional to how far away from his house you are. If the function is given as $B(d) = d^n$, where d is the distance away from his house, what limitations would you expect constant n to have?
   We'd expect n to have a *negative* value.

7. Your lifelong dream is to meet Lady Gaga in person. After seeing her in concert, you're 100% certain this dream will come true, but for every year that you don't see her live in concert, your half as sure that it'll really happen. Come up with a function that describes the yearly decay of your lifelong dream.
   $D(t) = 100 \times (\frac{1}{2})^t$ where $D(t)$ is the certainty at which you believe your dream will come true, and t is the number of years that pass after *not* seeing Lady Gaga in concert.

8. The potential energy of an object is given by $PE = mgh$. If the object is thrown and its height h is a function of time such that $h(t) = -0.6t^2 + 3t$, what is the potential energy of the object in terms of time?
   $PE = mgh(t) = mg(-0.6t^2 + 3t)$.

9. Using the function you came up with, at which point in time is PE the highest? What is this value in terms of m and g?
   $t = 2.5$, $PE = mgt(3 - 0.6t) = 3.75mg$.

10. If the overall energy of the object is the sum of its kinetic and potential energy ($KE = \frac{1}{2}mv^2$), and the velocity v of the object is a function of time such that $v = \frac{h}{t}$, what is the overall energy of the object in terms of m, g, and t?
    $E = mgt(3 - 0.6t) + \frac{1}{2}m(3 - 0.6t)^2$.