Convergent and Bounded Sequence- Answer Key

1. Find the bounds of the sequence $a_n = 3 - \frac{1}{n}$.
   $2 \leq a_n < 3$

2. Find the bounds of the sequence $a_n = 3n^2 - n$.
   The sequence is unbounded above and the lower bound is 2.

3. Does the sequence $a_n = \frac{3}{n^2}$ converge? Why?
   Yes, because the sequence in decreasing and bounded above by 3.

4. Does the sequence $a_n = \frac{n+1}{n^2}$ converge? Why?
   Yes, because the sequence in decreasing and bounded above by 2.

5. Does the sequence $a_n = \left(\frac{e^n}{n}\right)^2$ converge? Why?
   No, since $\lim_{n\to\infty} a_n = \infty$.

6. Does the sequence $a_n = \frac{(-2)^{n-1}}{n^2}$ converge? Why?
   Yes, since $\lim_{n\to\infty} |a_n| = 0$.

7. Does the sequence $a_n = \frac{an^2-2}{b^2n-n^2}$ converge? Why?
   Yes, since $\lim_{n\to\infty} a_n = -a$.

8. Does the sequence $a_n = (-2)^{n-1}$ converge? Why?
   No, since $\lim_{n\to\infty} |a_n|$ does not exist.

9. Does the sequence $a_n = \frac{\cos n}{n}$ converge? Why?
   Yes, since $\frac{-1}{n} \leq \frac{\cos n}{n} \leq \frac{1}{n}$ and by sandwich theorem $\lim_{n\to\infty} a_n = 0$.

10. Does the sequence $a_n = \frac{n}{\ln n}$ converge? Why?
    No since $\lim_{n\to\infty} a_n = \infty$.
