Continuity of Functions Worksheet

1. For $f(x) = 3x - 4$, when $|x - 2| < 0.1$ implies $|f(x) - 2| < 0.5$, identify c, ϵ and δ used in the definition of continuity.

2. Give an example of a function which is continuous on $[0, 1)$ but discontinuous on $(0, 1]$.

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4. Prove that $f(x) = x^2$ is continuous at $x = 0$.

5. Prove that $f(x) = \frac{1}{x}$ is continuous at $x = 1$.

6. For $f(x) = e^x$, find the largest δ such that $|x| < \delta$ implies $|f(x) - f(0)| < 0.01$.

7. Prove that $f(x) = \begin{cases} 0 & \text{if } x < -1; \\ 1 & \text{if } x \geq -1. \end{cases}$ is discontinuous at $x = -1$.

8. For $f(x) = 5x - 2$, find the largest δ such that $|x - 1| < \delta$ implies $|f(x) - f(1)| < 0.5$.

9. For $f(x) = x^2 + 1$, find the largest δ such that $|x - 1| < \delta$ implies $|f(x) - f(1)| < 0.1$.

10. Find the points of discontinuities for $f(x) = \begin{cases} -1 & \text{if } x \text{ is rational;} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$
